Lecture 03
Data Operations

CS213 – Intro to Computer Systems

Branden Ghena – Winter 2025

Slides adapted from:
St-Amour, Hardavellas, Bustamente (Northwestern), Bryant, O'Hallaron (CMU), Garcia, Weaver (UC Berkeley)

Northwestern

Administrivia

- You should all have access to Piazza and Gradescope
  - Contact me via email immediately if you don't!!

- Office hours are now running
  - See Canvas homepage for office hours times
  - Mostly in-person with a few online hours
    - Online uses gather.town (Room B)
  - Office hours queue on the Canvas homepage when things get busy
  - **Note:** no office hours next Monday for MLK Day

Administrivia

- Homework 1 due this week Thursday
  - Submit on Gradescope
- Pack Lab should be out later today!
  - Sometime this evening
- Pack Lab partnership survey on Piazza
  - If you want a partner but don’t know who you want to work with
  - I’ll pair people from the list right after I post the lab
- You’ll do Pack Lab on one of the EECS servers
  - Usually we use Moore, but any EECS server should be fine for this lab
  - SSH + Command Line interface
  - See Piazza post with some details on accessing the servers

Today's Goals

- Finish encodings thoughts from last time
- Explore operations we can perform on integers and more generally on binary numbers
- Understand the edge cases of those operations

Outline

- Binary and Hex
- Memory
- Encoding
- Integer Encodings
    - Signed Integers
    - Converting Sign
    - Converting Length
- **Other encodings**

Big Idea: What do bits and bytes ***mean*** in a system?

- The answer is: it depends!

- Depending on the context, the bits **`11000011`** could mean
  - The number 195
  - The number -61
  - The number -19/16
  - The character '├'
  - The **`ret`** x86 instruction

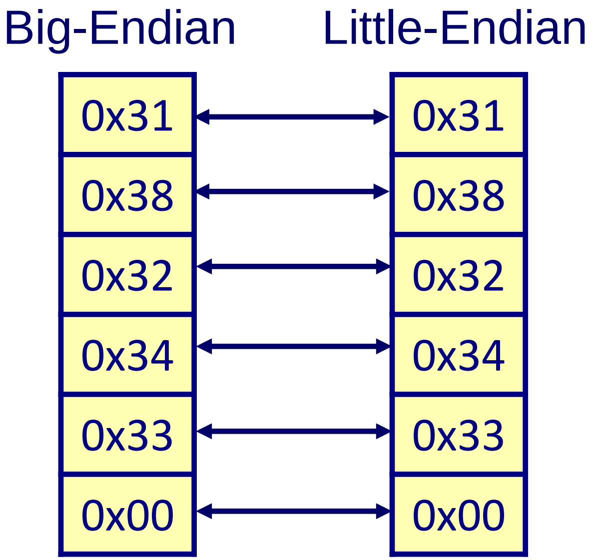
- You have to know the context to make sense of any bits you have!
  - Looking at the same bits in different contexts can lead to interesting results
  - Information = bits + context!

- An ***encoding*** is a set of rules that gives meaning to bits

Encoding strings (The C way)

- Represented by array of characters
  - Each character encoded in ASCII format
  - NULL character (code 0) to mark the end
- Compatibility
  - Byte ordering not an issue (data all single-byte!)
  - ASCII text files generally platform independent
    - Except for different conventions of line termination character(s)!

```
char S[6] = "18243";
```



Encoding color

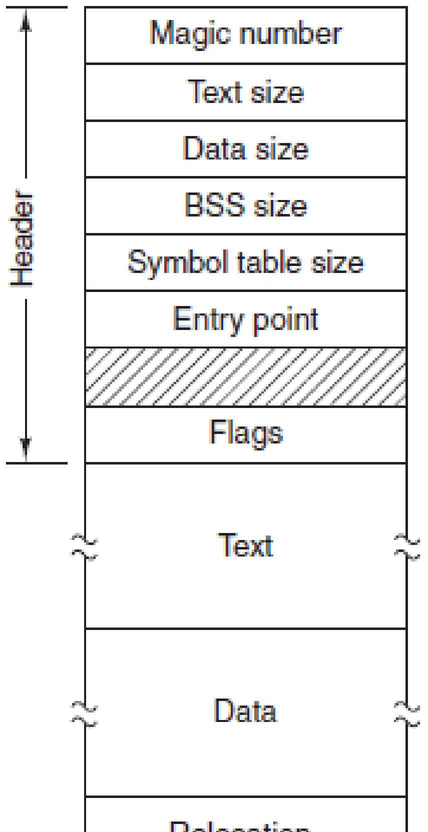
- RGB colors
  - 3-byte values
  - First byte is Red, then Green, then Blue
- Usually specified in hexadecimal
  - #FF0000 -> maximum red, zero green or blue
  - #4E2A84 -> 1/4 red, 1/8 blue, 1/2 green (Northwestern Purple)
- 2^{24} possible colors = 16777216 colors

Interpreting file contents

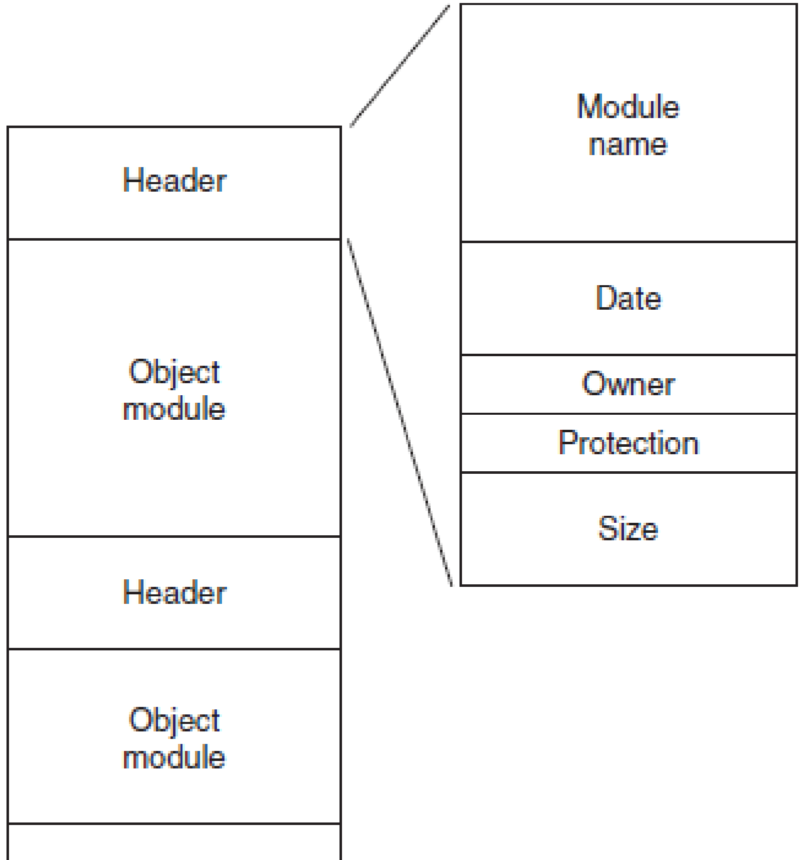
- Collections of data
  - Usually in permanent storage on your computer
- Regular files
  - Arbitrary data
  - Think of as a big array of bytes
- Non-regular files would be directories, symbolic links, or other less used things

What about different types of regular files?

- Text files versus Executables versus Tar files
  - All just differing patterns of bytes!
  - It really is just all data. The meaning is in how you interpret it.



Executable File



Archive (tar)

Identifying regular files

- **`file`** in Linux command line can help determine the type of a file
  - https://github.com/file/file

```
arguments  arguments.c
[brghena@ubuntu code] $ file arguments.c
arguments.c: C source, ASCII text
[brghena@ubuntu code] $ file arguments
arguments: ELF 64-bit LSB shared object, x86-64, version 1 (SYSV), dynamically linked, interpreter /lib64
/ld-linux-x86-64.so.2, BuildID[sha1]=8731c4961d371f4989cd1b056f796ad54b711e6f, for GNU/Linux 3.2.0, not s
tripped
[brghena@ubuntu code] $ file ./
./: directory
[brghena@ubuntu code] $ file ~/scratch/GlobalProtect_UI_deb-5.1.0.0-101.deb
/home/brghena/scratch/GlobalProtect_UI_deb-5.1.0.0-101.deb: Debian binary package (format 2.0), with cont
rol.tar.gz, data compression xz
```

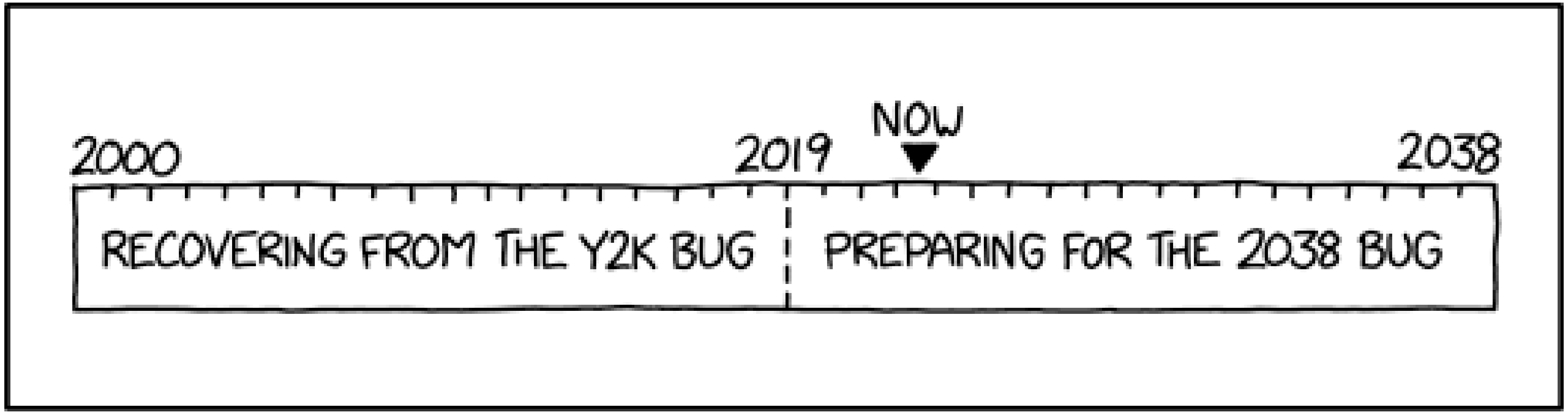
Encoding time

- Unix time:
  - 32-bit signed integer counting seconds elapsed since initial time
  - Initial time was January 1st at midnight UTC, 1970

- Current Unix time (as of last editing this slide): 1736874154
  - Negative numbers would mean times before 1970

- Problem: when does Unix time hit the maximum value?
  - 2147483647 seconds from January 1st 1970
  - Result: January 19th, 2038
  - This is the "Year 2038 Problem"

Bonus xkcd comic



REMINDER: BY NOW YOU SHOULD HAVE FINISHED YOUR Y2K RECOVERY AND BE SEVERAL YEARS INTO 2038 PREPARATION.

https://xkcd.com/2697/

Outline

- **Integer Operations**
  - **Addition**
  - Negation and Subtraction
  - Multiplication and Division

- Binary Operations
  - Boolean Algebra
  - Shifting
  - Bit Masks

C versus the hardware

- Operations you can perform on binary numbers have edge conditions
  - Usually going above or below the bit width

- If we say what happens in that scenario, it'll be what "the hardware" (i.e., a computer) does
  - In today's examples, pretty much every computer does the same thing

- That is not the same as what C does
  - Unclear choices are left as: **UNDEFINED BEHAVIOR** 😱
  - Which is to say, the compiler can make any choice it wants

Unsigned Addition

- Like grade-school addition, but in base 2, and ignores final carry
  - If you want, can do addition in base 10 and convert to base 2. Same result! But here we're going to understand what the hardware is doing.
- **Example: Adding two 4-bit numbers**

$$\begin{array}{r} {\scriptstyle 1\,1\,1} \\ \texttt{0101} \\ +\ \texttt{0011} \\ \hline \texttt{1000} \end{array}$$

  - $\mathbf{5_{10} + 3_{10} = 8_{10}}$ ✓

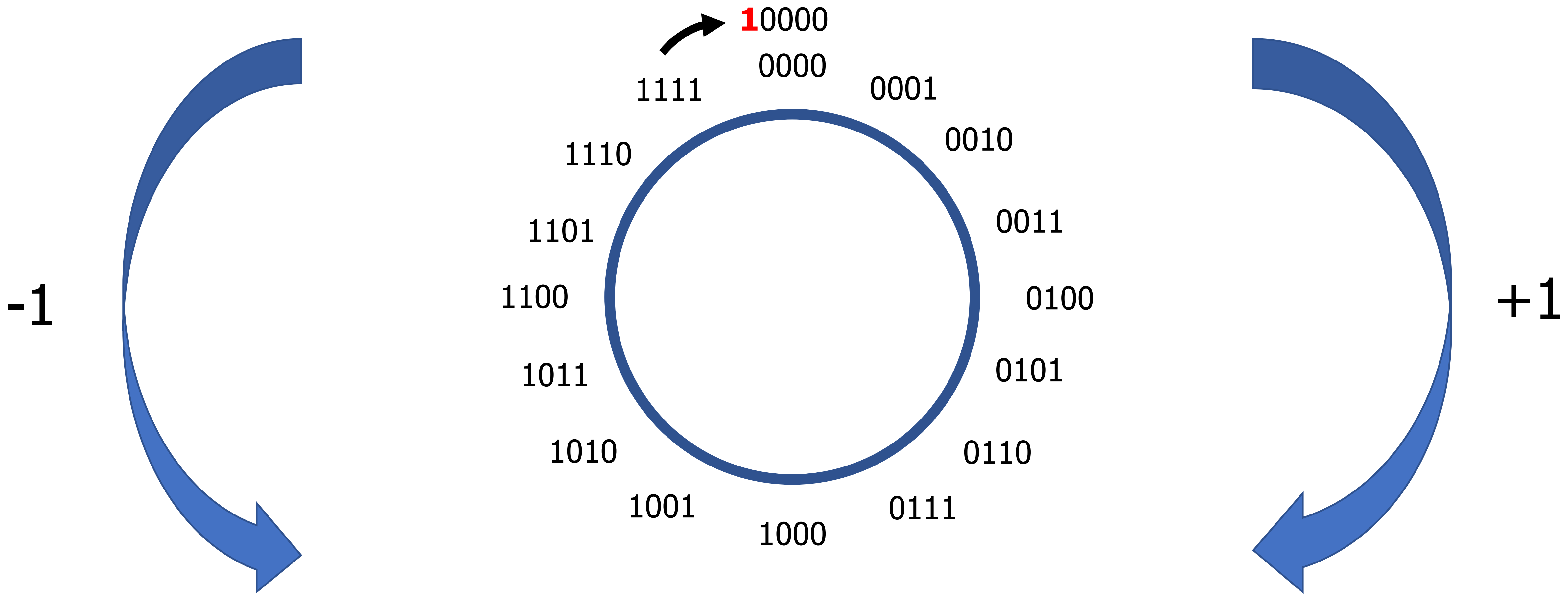
Unsigned Addition and Overflow

- What happens if the numbers get too big?
- **Example: Adding two 4-bit numbers**

```
 1 1 1 1
   1101
+  0011
  -----
  10000
```

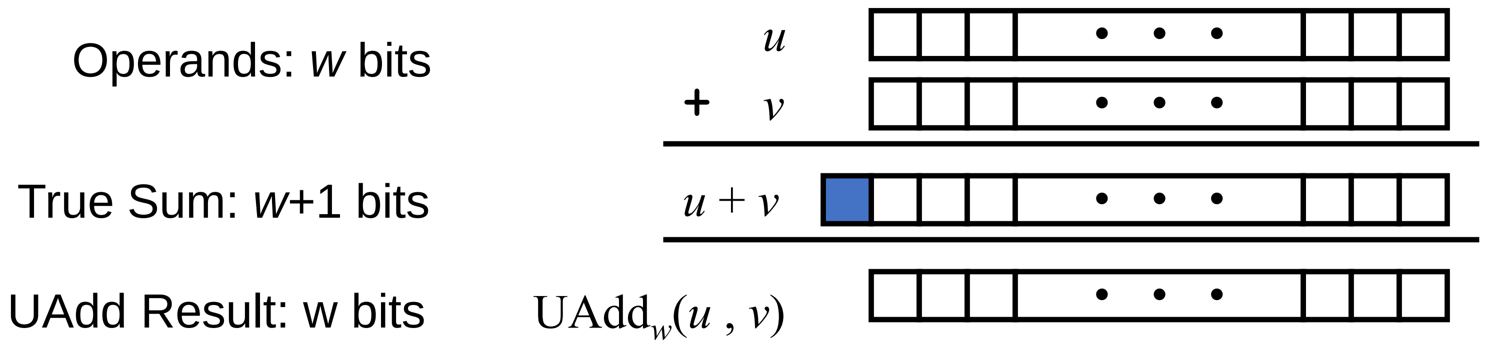
- **13_{10} + 3_{10} = 16_{10}**
  - Too large for 4 bits! Overflow
  - Result is the 4 least significant bits (all we can fit): so 0_{10}
  - Truncate most-significant bits that do not fit
    - Gives us modular (= modulo) behavior: 16 modulo 2^4 = 0

Modulo behavior in binary numbers



Unsigned addition is modular

- Implements modular arithmetic
  - $\mathrm{UAdd}_w(u\ ,\ v) \quad = \quad (u + v) \bmod 2^w$
- Need to drop carry bit, otherwise results will keep getting bigger
  - Example in base 10: $80_{10} + 40_{10} = 120_{10}$ (2-digit inputs become a 3-digit output!)



- Warning: C does not tell you that the result had an overflow!
  - **Unsigned** addition in C silently truncates most-significant bits beyond the limit

Signed (2’s Complement) Addition

- Works exactly the same as unsigned addition!
  - Just add the numbers in binary, and the result will work out

- Signed and unsigned sum have the exact same bit-level representation
  - Computers use the same machine instruction and the same hardware!
  - That’s a big reason 2’s complement is so nice! Shares operations with unsigned

Signed addition example

- Same addition method as unsigned
- **Example: Adding two 4-bit signed numbers**

```
   1 1
  1011   (-8 + 3 = -5)
+ 0011   (        +3)
  ----
  1110   (-8 + 6 = -2)
```

- **$-5_{10} + 3_{10} = -2_{10}$ ✓**

Combining negative and positive numbers

- Overflow sometimes makes signed addition work!
- **Example: Adding two 4-bit signed numbers**

```
 1 1 1 1
  1101   (-8 + 5 = -3)
+ 0011   (        +3)
 -----
 10000
```

- **$-3_{10} + 3_{10} = 0_{10}$**
  - Too large for 4 bits! Drop the carry bit
  - Result is what we expect as long as we truncate

Overflow: hardware vs C standard

- Hardware implementations for unsigned and signed addition are the same
  - Both implement truncation of overflowing bits, leads to modular arithmetic

- Unsigned overflow in C is defined as modular arithmetic

- Signed overflow in C is **UNDEFINED BEHAVIOR**
  - Compiler *probably* does modular result
  - But there are no promises about this and it can make *assumptions*
  - So don’t rely on it
  - Generally: overflow is bad and is to be avoided!

Overflow in the real world

- In Switzerland, there’s a rule that trains are not allowed to have exactly 256 axles

3.7.4 Zugbildung

Um das ungewollte Freimelden von Streckenabschnitten durch das Rückstellen der Achszähler auf Null und dadurch Zugsgefährdungen zu vermeiden, darf die effektive Gesamtachszahl eines Zuges nicht 256 Achsen betragen.

"To avoid falsely signalling a section of track as clear by resetting the axle counter to zero, and thus to avoid [collisions], the total number of axles in a train must not equal 256."

Video games bugs can be exploited

- Dream Devourer
  - Special boss in the Nintendo DS edition
- Wanted to make it even more challenging
  - ~32000 hit points
  - Takes *forever* to defeat
- Hit points stored as a 16-bit signed integer
  - Range: -32768 to +32767
- **How do speedrunners defeat the boss?**

Chrono Trigger signed overflow bug

- Solution: heal it
- Hit points go negative and it dies

Outline

- **Integer Operations**
  - Addition
  - **Negation and Subtraction**
  - Multiplication and Division

- Binary Operations
  - Boolean Algebra
  - Shifting
  - Bit Masks

Negating a number

- In C:
  - `x = -y;`
- Operation
  - Determine the negative, signed version of the number (two’s complement)
  - Hardware method: flip bits and add one
- Complement operator (~)
  - Flips all bits: zeros become a one and ones become a zero
  - ~0b1011 -> 0b0100

Negating via Complement & Increment

- Claim: The following is true for 2's complement
  - ~x + 1 == -x

`x`	1 0 0 1 1 1 0 1
`+ ~x`	0 1 1 0 0 0 1 0
`-1`	1 1 1 1 1 1 1 1

- Complement
  - Observation: ~x + x == $1111\ldots11_2$ == -1

- Increment
  - $\sim x + 1 == \sim x \; \boldsymbol{+ x} \; \boldsymbol{- x} + 1 == \cancel{-1} - x + \cancel{1} == -x$

- Example, 4 bits: $6_{10} = 0110_2$
  - Complement: $1001_2 \rightarrow$ Increment = $1010_2 = -8 + 2 = -6_{10}$

Subtraction in two’s complement

- Subtraction becomes addition of the negative number
  - 5 – 3 = 5 + -3 = 2

- Both unsigned and signed subtraction
  - Convert subtractor to its two’s complement negative form
    - i.e., negate it
  - Then do addition
  - Treat result as an unsigned number

```
  11 1
   0101    (+5)
+  1101    ( -3)
   ----
  10010
```

C rules vs hardware rules

- Exact same overflow rules apply

- Unsigned subtraction overflow can wrap below zero to make a large number
  - Modular arithmetic

- Signed subtraction overflow is **UNDEFINED BEHAVIOR**
  - And therefore should not be trusted

Break + practice

- Adding two 8-bit binary numbers:
  - Also determine the decimal version of the result

```
  00010101
+ 10110001
 _________
```

Break + practice

- Adding two 8-bit binary numbers:
  - Also determine the decimal version of the result

```
  1 1    1
  00010101
+ 10110001
  --------
  11000110
```

	Unsigned encoding		Signed encoding
00010101	16+4+1 = 21		16+4+1 = 21
10110001	128+32+16+1 = 177	OR	-128+32+16+1 = -79
11000110	128+64+4+2 = 198		-128+64+4+2 = -58

Break + practice

- Adding two 8-bit binary numbers:
  - Also determine the decimal version of the result

```
    1 1     1
   00010101
 + 10110001
 ----------
   11000110
```

	Unsigned encoding		Signed encoding
00010101	16+4+1 = 21		16+4+1 = 21
+ 10110001	128+32+16+1 = 177	OR	-128+32+16+1 = -79
11000110	128+64+4+2 = 198		-128+64+4+2 = -58

What about unsigned subtraction 21-79?

That would treat the result as unsigned, with the value 198

Modular arithmetic in action

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Multiplication

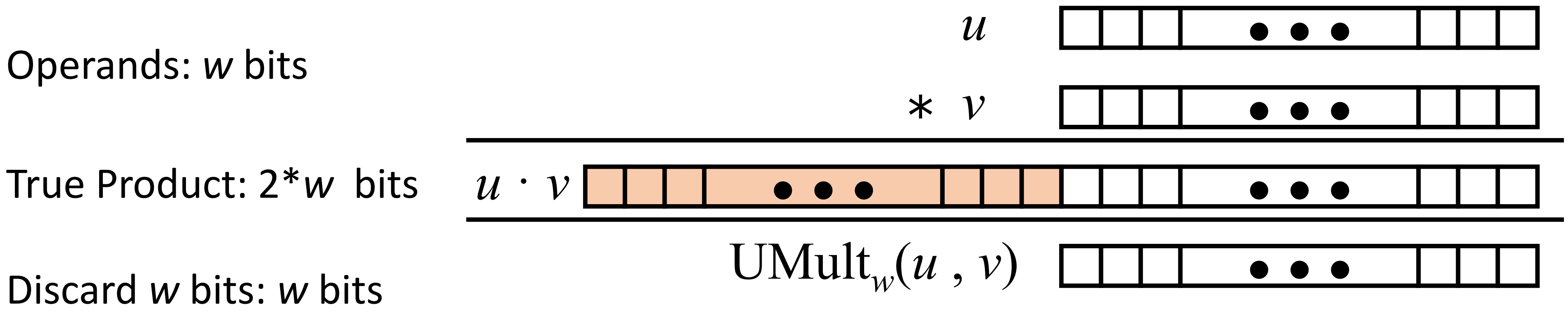
- Goal: Compute the Product of two ***w***-bit numbers *x*, *y*
  - Either signed or unsigned

- But, exact results can be bigger than ***w*** bits
  - Double the size (2***w***), in fact!

  - Example in base 10: $50_{10} * 20_{10} = 1000_{10}$
    - (2-digit inputs become a 4-digit output!)

- As with addition, result is truncated to fit in ***w*** bits
  - Because computers are finite, results can't grow indefinitely

Unsigned Multiplication



- **Standard Multiplication Function**
  - Equivalent to grade-school multiplication
  - But ignores most significant w bits of the result
  - As a person, we can do base 10 multiplication, convert to base 2, then truncate

- Implements modular arithmetic like addition does

  $\text{UMult}_w(u , v) \;=\; (u \cdot v) \bmod 2^w$

Unsigned multiplication

- **Example: Multiplying two 4-bit numbers**

```
      0010
  x   0101
  --------
      0010
     00000
    001000
+  0000000
----------
   0001010
```

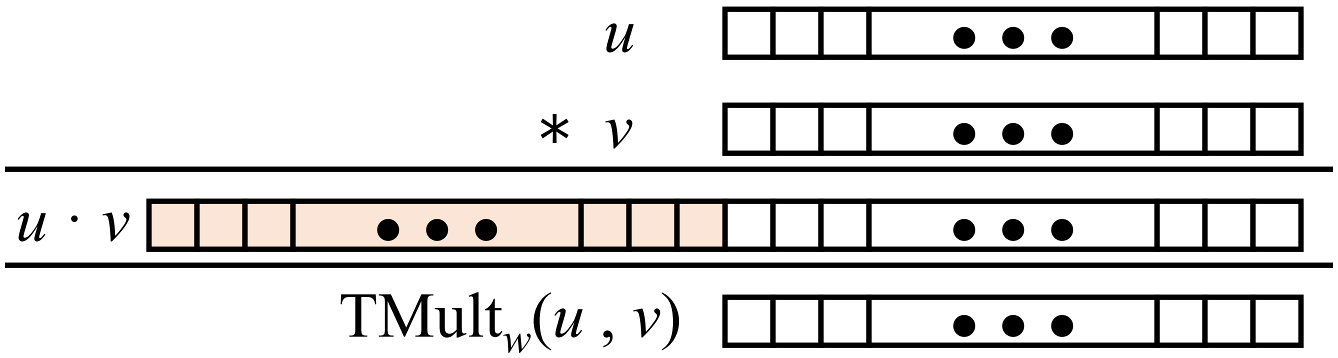
$$\mathbf{2_{10} * 5_{10} = 10_{10}}\ ✓$$

Signed (2’s Complement) Multiplication

Operands: *w* bits

True Product: 2**w* bits

Discard *w* bits: *w* bits



- **Standard Multiplication Function**
  - Ignores most significant *w* bits
  - Lower bits still give the correct result
    - So we can use same machine instruction for both!
    - Again, that’s one reason why 2’s complement is so nice

- **In C, signed overflow is undefined**
  - ...but probably you’ll see the two’s complement behavior

Signed multiplication

- **Example: Multiplying two's complement 5-bit numbers**

```
     11110   -2
  x  00011    3
     -----
     11110
 +  111100
  --------
  ~~10~~11010
```

What are these two 5-bit numbers?

What is the result of this addition?

$-2_{10} * 3_{10} = -6_{10}$ ✓

What about divide?

- Annoying operation, not going to discuss in this class
  - Similar to long division process
  - Tedious and complicated to get right

- I’ve worked on computers that don’t have hardware support for division at all!!

- Important thing to remember is that integers don’t have fractional parts
  - In C: `1/2 == 0`
  - We’ll need a different encoding for fractional numbers: floating point

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Boolean algebra

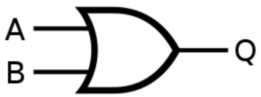
- You’ve programmed with **`and`** and **`or`** in earlier classes
  - Written **`&&`** and **`||`** in C and C++

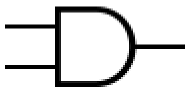
- **Boolean algebra is a generalization of that**
  - A mathematical system to represent logic (propositional logic)
  - 2 truth values: true = **`1`**, false = **`0`**
  - Operations: and **`&`**, or **`|`**, not (or complement) **`~`**

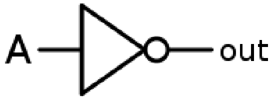
Performing Boolean algebra

- **Follow the rules for each operation to compute results**
  - Rules are like those you know from programming

- OR: | AND: &   NOT: ~   1: True   0: False





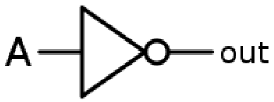


$$(1 \mid 0)\ \&\ 0 \longrightarrow 1\ \&\ 0 \longrightarrow 0$$

$$(1\ \&\ 1)\ \&\ {\sim}(0 \mid 0) \longrightarrow 1\ \&\ {\sim}(0) \longrightarrow 1\ \&\ 1 \longrightarrow 1$$

Truth tables for Boolean algebra

- For each possible value of each input, what is the output
  - Column for each input
  - Column for the output operation

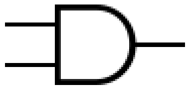


~A

A	~A
0	1
1	0

A | B

A	B	A | B
0	0	0
0	1	1
1	0	1
1	1	1



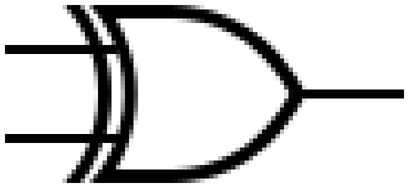
A & B

A	B	A & B
0	0	0
0	1	0
1	0	0
1	1	1

Exclusive Or (xor)

A ^ B

A	B	A ^ B
0	0	0
0	1	1
1	0	1
1	1	0



- An operation you likely haven’t used before:
  - Xor - either A or B, but not both
  - ^ symbol in C
- We can build Xor out of &, |, and ~
  - A^B = (~A & B) | (A & ~B)
    - (exactly one of A and B is true)
  - A^B = (A | B) & ~(A & B)
    - (either is true but not both are true)
  - The two definitions are equivalent
    - Produce the same Truth Table

Practice problem

(A & B) | B

A	B	(A&B)|B
0	0	
0	1	
1	0	
1	1	

Practice problem

(A & B) | B

A	B	(A&B)|B
0	0	0
0	1	1
1	0	0
1	1	1

Practice problem

(A & B) | B

A	B	(A&B)|B
0	0	0
0	1	1
1	0	0
1	1	1

This is equivalent to B

(A has no influence on the solution)

Generalized Boolean algebra

- Boolean operations can be extended to work on collections of bits (i.e., bytes)

- Operations are applied one bit at a time: ***bitwise***

```
   01101001      01101001      01101001
 & 01010101    | 01010101    ^ 01010101    ~ 01010101
   --------      --------      --------      --------
   01000001      01111101      00111100      10101010
```

- All of the properties of Boolean algebra still apply
  - Relationships between operations, etc.

- Bitwise operations are usable in C: **&**, **|**, **~**, **^**
  - Can operate on any integer type (long, int, short, char, signed or unsigned)

Warning: bitwise operations are NOT logical operations

- Logical operations in C: **||**, **&&**, **!** (logical Or, And, and Not)
  - Only operate on a single bit
    - View 0 as "False"
    - View *anything nonzero* as "True"
    - Always return 0 or 1
  - Short-circuit evaluation: only checks the first operand if that is sufficient

- Examples
  - !0x41 -> 0x00 !0x00 -> 0x01 !!0x41 -> 0x01
  - 0x59 && 0x35 -> 0x01
  - (p != NULL) && *p (short circuit evaluation avoids null pointer access)

- Don't confuse the two!! It's a common C mistake

Break + Practice: C example of bitwise operators

```
unsigned char x = 13;
unsigned char y = 11;
unsigned char z = x & y;
```

- What decimal value is in `z` now?
  - Remember: `unsigned char` is an 8-bit value

Break + Practice: C example of bitwise operators

```
unsigned char x = 13;
unsigned char y = 11;
unsigned char z = x & y;
```

- What decimal value is in `z` now?
  - Remember: `unsigned char` is an 8-bit value

  - x: 0b00001101
  - y: 0b00001011
  - z: 0b00001001 -> 9

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- Integer Operations
  - Addition
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  - Multiplication and Division
- **Binary Operations**
  - Boolean Algebra
  - **Shifting**
  - Bit Masks

Left Shift: `x << y`

- Shift bit-vector `x` left by `y` positions
  - Throw away extra bits on left
  - Fill empty bits with 0
    - Same behavior for signed or unsigned

Argument `x`	`00000010`
`<< 3`	~~`000`~~`00010`*`000`*

Argument `x`	`10100010`
`<< 3`	~~`101`~~`00010`*`000`*

- Equivalent to multiplying by 2^y
  - And then taking modulo (i.e. truncating overflow bits)

- Undefined behavior in C when:
  - `y < 0`, or `y ≥ bit_width(x)`
  - Also when some non-0 bits get shifted off (*probably* they get truncated)

Right Shift: `x >> y`

- Shift bit-vector x right y positions
  - Throw away extra bits on right
- But how to fill the new bits that open up?
  - Will depend on signed vs unsigned

Argument `x`	01100010
Logi. >> 2	00011000
Arith. >> 2	00011000

Argument `x`	10100010
Logi. >> 2	00101000
Arith. >> 2	11101000

- Unsigned: Logical shift
  - Always fill with 0's on left
- Signed: Arithmetic shift
  - Replicate most significant bit on left
  - Necessary for two's complement integer representation (sign extension!)
- Undefined behavior in C when:
  - `y < 0,    or    y ≥ bit_width(x)`

Practice shifting in C

```c
unsigned char x = 0b10100010;
x << 3 = ?  0b00010000
```

Steps:
0b10100010**000**
0b~~101~~00010**000**

```c
unsigned char x = 0b10100010;
x >> 2 = ?   0b00101000
```

Steps:
0b**00**10100010
0b**00**101000~~10~~

```c
signed   char x = 0b10100010;
x >> 2 = ?   0b11101000
```

Steps:
0b**11**10100010
0b**11**101000~~10~~

Note:
GCC supports the prefix **0b** for binary literals (like **0x**… for hex) directly in C.
This is not part of the C standard! It may not work on other compilers.

Concept: Not all operations are equally expensive!

- Some operations are pretty simple to perform in hardware
  - E.g., addition, shifting, bitwise operations
  - Also true of doing the same by hand on paper

- Others are much more involved
  - E.g., multiplication, or even more so division
  - Consider long multiplication / long division; quite tedious!
  - Hardware is not doing the exact same thing, but similar principle

- For best performance: swap expensive operations with simple ones!
  - Doesn’t work in all cases, but often does when mult/div by constants

Compilers automatically chose the best operations

- Should you use shifts instead of multiply/divide in your C code?
  - **NO**

- Just write out the math
  - Math is more readable if that's what you meant
  - **Compiler** automatically converts code to get best performance

- These two mean the same thing, but one is way more understandable
  - `int x = y * 32;`
  - `int x = (y << 5);`

Outline

- Integer Operations
  - Addition
  - Negation and Subtraction
  - Multiplication and Division

- **Binary Operations**
  - Boolean Algebra
  - Shifting
  - **Bit Masks**

Bit Masking

- How do you manipulate certain bits within a number?

- Combines some of the ideas we've already learned
  - ~, &, |, <<, >>

- Steps
  1. Create a "bit mask" which is a pattern to choose certain bits
  2. Use & or | to combine it with your number
  3. Optional: Use >> to move the bits to the least significant position

How to operate on bits

- Selecting bits, use the AND operation
  - 1 means to select that bit
  - 0 means to not select that bit

Select bottom four bits:

```
num & 0x0F
```

- Writing bits
  - Writing a one, use the OR operation
    - 1 means to write a one to that position
    - 0 is unchanged
  - Writing a zero, use the AND operation
    - 0 means to write a zero to that position
    - 1 is unchanged

Set 6^{th} bit to one:

```
num | (1 << 6)
num | (0b01000000)
```

Clear 6^{th} bit to zero:

```
num & (~(1 << 6))
num & (~(0b01000000))
num & (0b10111111)
```

Example: swap nibbles in byte

- Nibble - 4 bits (one hexit)
  - Input: 0x4F -> Output 0xF4

  - Method:
    - 1. Shift and select upper four bits
    - 2. Shift and select lower four bits
    - 3. Combine the two nibbles

What are the values of the new upper bits?

Unsigned -> Will be zero

```
uint8_t lower = input >> 4;
uint8_t upper = input << 4;
uint8_t output = upper | lower; // combines two halves
```

Shifting implicitly zero'd out irrelevant bits.
Otherwise we would have needed an & operation too.

Example: selecting bits

- Select bits 2 and 3 from a number

```
Input: 0b01100100
 Mask: 0b00001100
```

```
   0b01100100
 & 0b00001100
   ----------
   0b00000100
```

Finally, shift right by two to get the values in the least significant position:

```
0b00000001
```

In C:

```
result = (input & 0x0C) >> 2;
```

Outline

- Integer Operations
  - Addition
  - Negation and Subtraction
  - Multiplication and Division
- Binary Operations
  - Boolean Algebra
  - Shifting
  - Bit Masks

Outline

- Dividing with bit shift

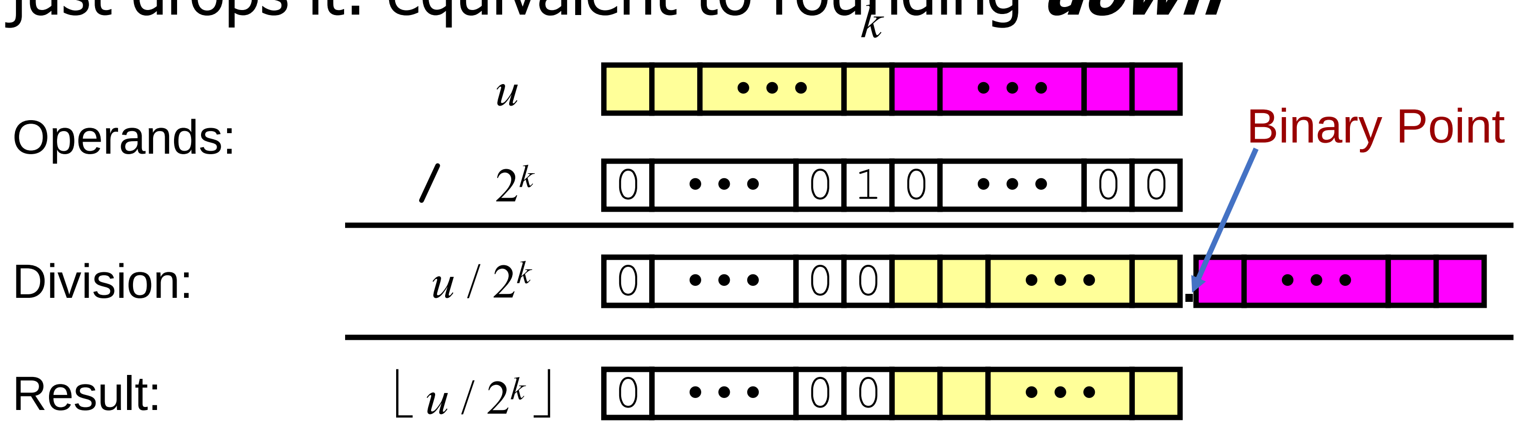
- Bonus material isn't required and won't be on an exam
  - Unless it becomes main lecture material in a different lecture

- Usually the material is just for students who want more depth
  - As is the case here

Unsigned Power-of-2 Divide with Right Shift

$\lfloor x \rfloor$: round x down
$\lceil x \rceil$: round x up

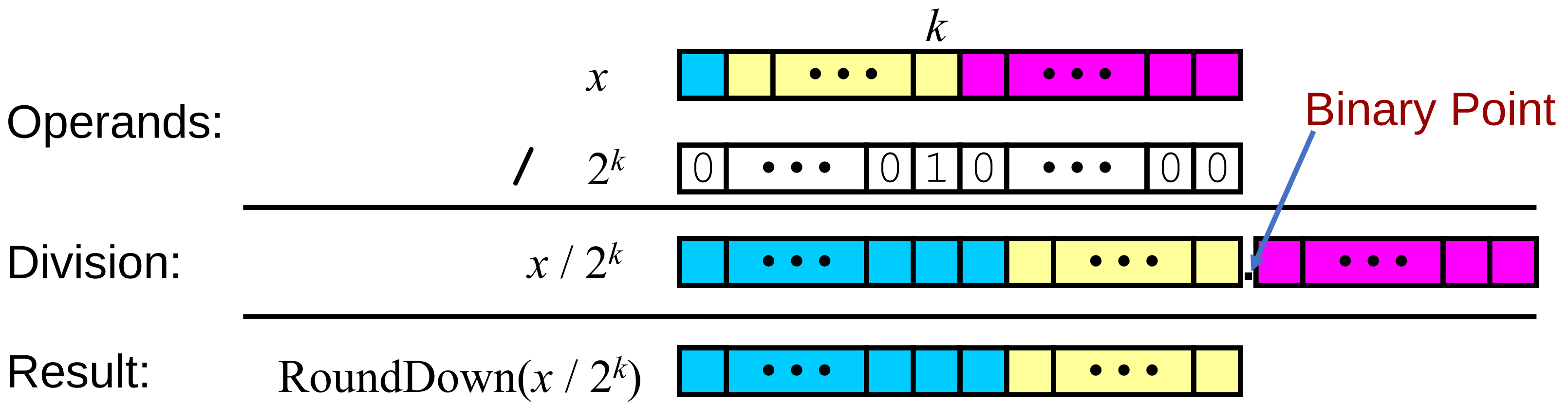
- **Quotient of unsigned by power of 2**
  - **`u >> k`** gives $\lfloor \mathbf{u} \;/\; \boldsymbol{2^k} \rfloor$
  - Uses logical shift
  - Pink part would be remainder / fractional part (right of the point)
    - Shift just drops it: equivalent to rounding ***down***



	Division	Computed	Hex	Binary
`x`	15213	15213	`3B 6D`	`00111011 01101101`
`x >> 1`	7606.5	7606	`1D B6`	`00011101 10110110`
`x >> 4`	950.8125	950	`03 B6`	`00000011 10110110`
`x >> 8`	59.4257813	59	`00 3B`	`00000000 00111011`

Signed Power-of-2 Divide with Shift (Almost)

- **Quotient of signed by power of 2**
  - `x >> k` gives $\lfloor \mathbf{x}\ /\ 2^k \rfloor$
  - Uses arithmetic shift
  - Also rounds down, again by dropping bits
    - But signed division should round ***towards 0!*** (that's its math definition)
    - That means rounding ***up*** for negative numbers!

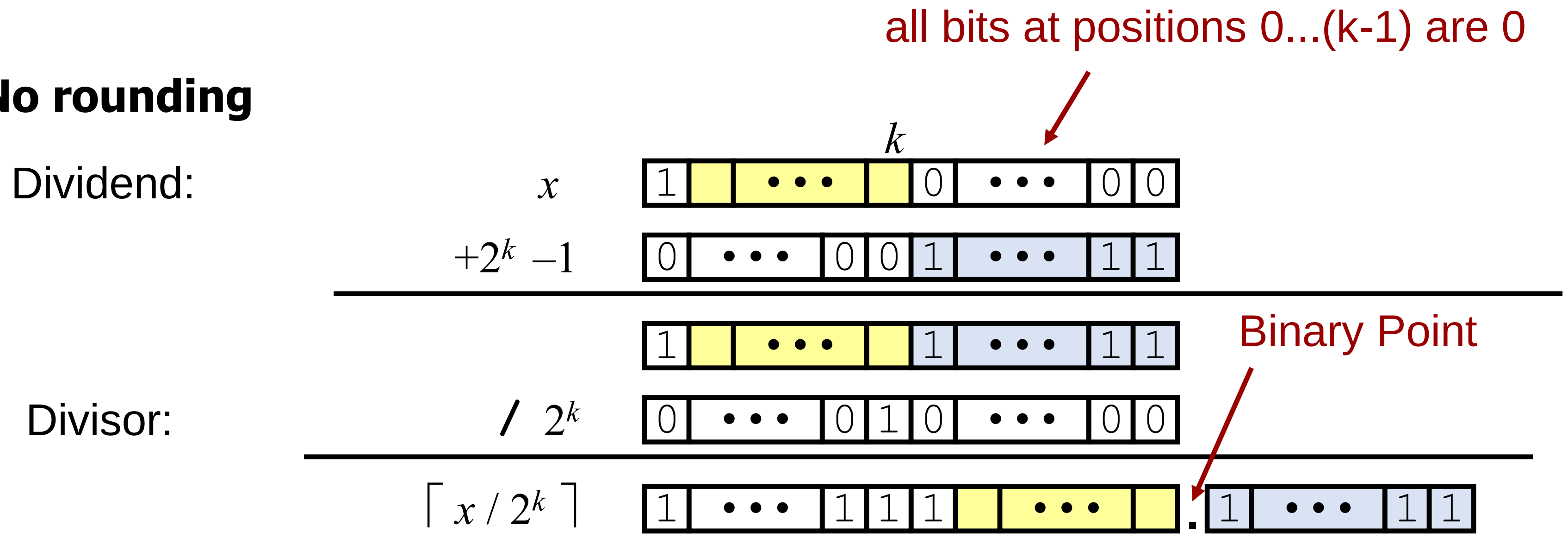


- **Example, 4 bits: -6 / 4 = -1.5 (should round towards 0, to -1)**
  - $1010_2 >> 2 = \mathit{11}10_2 = -2_{10}$
  - Rounds the wrong way!

Correct Signed Power-of-2 Divide

- Want $\lceil$ **x / 2^k** $\rceil$ (round towards 0)
  - Math identity: $\lceil$**x / y**$\rceil$ **=** $\lfloor$ **(x + y - 1) / y** $\rfloor$
  - Compute negative case as $\lfloor$ **(x+2^k–1) / 2^k** $\rfloor$ → gets us correct rounding!
  - Computing both cases in C: `(x<0 ? (x + (1<<k)-1) : x) >> k`
    - Biases dividend toward 0

- **Case 1: No rounding**

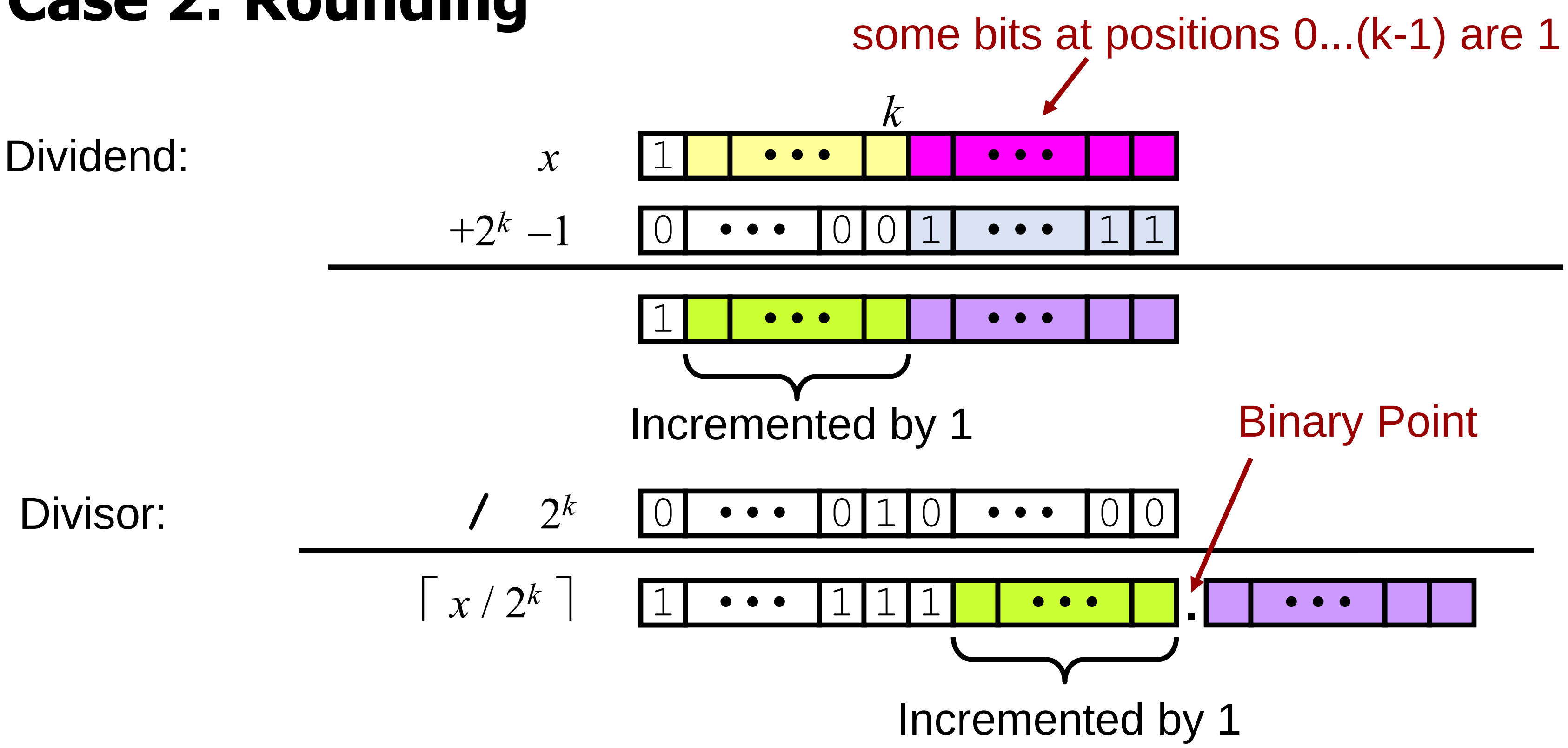


Biasing has no effect; all affected bits are dropped

- **Example, 4 bits: -8 / 2^2 = -2 bias = (1<<2)-1 = 3**
  - (1000 + 0011) >> 2 = 1011 >> 2 = 1110 = -2$_{10}$ (correct, no rounding)

Correct Signed Power-of-2 Divide (Cont.)

Case 2: Rounding



Biasing adds 1 to final result; just what we wanted

- **Example, 4 bits: -6 / 2^2 = -1** **bias = (1<<2)-1 = 3**
  - (1010 + 0011) >> 2 = 1101 >> 2 = *11*11 = -1_{10} (correct, rounds towards 0)
- **Compiler does that for you (but you need to be able to read it!)**